PSF Whisker Requirements for the Dark Energy Survey

Stephen Kent

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SUMMARY

Over the area of a single CCD, the residual PSF whisker amplitude shall be no more than $0.04''$ after removal of a linear fit in both x and y.

BACKGROUND

Weak lensing measurements require that the PSF shape of an unresolved source be measured to high accuracy. The purpose of this research note is to quantify what is meant by "high accuracy."

Weak lensing causes the shapes of intrinsically round objects to be distorted slightly such that they appear to be slightly elliptical. While no single object is intrinsically round, if the intrinsic shapes of galaxies are uncorrelated with one another, one can average the apparent shapes of many thousands of such objects to extract an apparent distortion that is attributed to weak lensing. However, the measured shapes of galaxies will include a component due to the point spread function (PSF) of the combined telescope, atmosphere, and instrument that is correlated among galaxies. Removing this contribution requires careful measurement of the PSF, which is done using isolated stars in the same field.

First, begin with definitions of fundamental quantities that characterize weak lensing measurements. In the standard theory of conic sections, one parameterizes an ellipse in the following manner. Let

$$\begin{aligned} a &= \text{major axis}, \\ b &= \text{minor axis}, \\ \phi &= \text{position angle of major axis w.r.t. a fiducial } x \text{ axis.} \end{aligned}$$

Often one uses one or more of three derived parameters:

$$\begin{aligned} \epsilon &= 1 - b/a \text{ is the ellipticity}, \\ c &= \sqrt{a^2 - b^2} \text{ is the focus or ``whisker length''}, \\ e &= c/a = \sqrt{1 - (1-\epsilon)^2} \text{ is the eccentricity.} \end{aligned}$$

All parameters except ellipticity are well established in mathematics. The definition of ellipticity follows Hubble's 1936 convention. In optics, the eccentricity e is also referred to as the "conic constant".

Note that for small ellipticities, $e \approx \sqrt{2\epsilon}$.

The weak lensing community has established its own definitions and notations that are inconsistent with well established tradition; however, we plow ahead bravely, nonetheless.

The "ellipticity vector" is defined in terms of the second moments of a PSF as follows:

$$e_1 = (\langle x^2\rangle - \langle y^2\rangle)/(\langle x^2\rangle + \langle y^2\rangle),$$
$$e_2 = 2\langle xy\rangle/(\langle x^2\rangle + \langle y^2\rangle).$$

In terms of the conic parameters, we have:

$$\langle x^2\rangle + \langle y^2\rangle = a^2 + b^2,$$
$$e_2/e_1 = \tan 2\phi,$$
$$\sqrt{e_1^2 + e_2^2} = e^2/(2 - e^2) = c^2/(a^2 + b^2).$$

For small ellipticities, the length of the "ellipticity" vector is, in fact, the classical Hubble ellipticity ϵ (whew!).

Define an "ellipticity vector" $\vec{\epsilon}$ whose amplitude is the ellipticity ϵ and whose position angle is 2ϕ. This vector characterizes the "shear" in the shape of a galaxy induced by weak lensing. The reason for the factor 2 multiplying the position angle is that shear is a tensor quantity; rotation of an ellipse by π radians leaves the ellipse unchanged.

While the weak lensing signal itself is a dimensional quantity characterized, say, by the ellipticity vector, the PSF is a convolution of a number of contributions. Dimensionless parameters like ellipticity are not preserved during convolution; however dimensional parameters, particularly the "whisker length" c add in quadrature during convolution. For this reason, it is preferable to place requirements on whisker length, not PSF shape.

WEAK LENSING EXPERIMENT

We construct a very simple-minded weak lensing experiment that, nevertheless, conveys enough information to establish requirements on the optics.

Take a DES field of diameter 2.2^o. Divide the field into two regions, which are separated by half this - roughly 1^o. Measure the ellipticity vector $\vec{\epsilon}$ for each galaxy. Let $\vec{\gamma^j}$ be the average of the ellipticity vectors in region j. The contributions to γ from the shapes of inidividual galaxies will largely cancel out (assuming that they are uncorrelated), but the contribution of shear due to weak lensing, averaged over the region, will not cancel.

The covariance between the two regions is a measure of the correlation in the shapes of galaxies that are separated by 1^o. Let $\gamma_i^2 = \vec{\gamma^1} \cdot \vec{\gamma^2}$ be the amplitude of the dot product between the two regions for the i^{th} DES field. This parameter, for one field, gives one measure of the covariance on 1^o scales. We average this quantity over all $N_f \approx 5000$ DES fields:

$$\bar{\gamma^2} = \sum_{i=1}^{N_f} \gamma_i^2 / N_f.$$

This quantity gives an estimate of the true covariance, $\langle \gamma^2 \rangle$ on 1^o scales. The "error" in this estimate (i.e., the difference between the estimate and the true covariance) has three contributions:

1) "shape noise" - the fact that galaxies are intrinsically not round, but we expect their shapes to be uncorrelated, so averaging the shapes of thousands of galaxies ultimately gives a measured of the intrinsic cosmic shear;
2) "cosmic variance" - the fact that we only sample a finite number of more-or-less independent pieces of the universe;
3) systematic error introduced by imperfect correction of measurements for the PSF.

On 1^o scales, the first two contributions are comparable. [This result is contrary to common wisdom, I am told, but I don't know why I get a different result.] The "shape noise" σ_{shape} for one field is given approximately by $\sigma_{shape1field} \approx (0.32)^2 \times 2/(2N_{gal})$, where 0.32 is the typical ellipticity of a galaxy, N_{gal} is the number of galaxies per field, the factor 2 accounts for the fact that we divide the field in half, that we compute the difference in ellipticity between the two, and that a factor 2 comes in when computing the "error" in the variance. The shape noise is expected to be uncorrelated among fields, and thus the total error in the variance is given by $\sigma_{shape} \approx (0.32)^2 \times 2/(N_{gal}\sqrt{N_f})$. For a density of 10 galaxies arcmin^{-1}, we find $\sigma_{shape} \approx 2 \times 10^{-8}$.

Each DES field provides one measure of the covariance on 1^o scales. The total number of samples is N_f. The "error" in estimating the covariance, γ^2, from a large number of samples depends on the statistical properties of the shear field. For Gaussian statistics, the "error" in $\bar{\gamma^2}$ due to cosmic variance is roughly $\sigma_{CV} \approx \gamma^2/\sqrt{N_f}$. For $\gamma = 0.001$ (the approximate value on 1 degree scales) and $N_f = 5000$, the error in estimating γ^2 is thus $\sigma_{CV} = 1.4 \times 10^{-8}$.

An imperfectly calibrated PSF will also contribute to the estimated covariance. The calibration procedure, at a minimum has three features:

1) The PSF for each CCD is calibrated independently, using stars;
2) A linear fit as a function of CCD row and column is made to each of the e_1 and e_2 parameters measured for stars;
3) A principal components analysis (PCA) will be applied to the residual PSF shapes from a combination of many frames to better calibrate that portion of the PSF spatial variation that cannot be modelled with a linear function.

Let w be the uncalibrated residual"whisker" length for a stellar PSF, as defined above, after removal of the linear term in step (2). This parameter contains contributions from many causes, including the optics, telescope tracking error, wind shake, etc. These contributions all add in quadrature. Let g be the characteristic size of the smallest galaxy that will be used in the weak lensing analysis. In practice g is of order the total PSF size, about $0.9''$ FHWM. The induced ellipticity in a galaxy image is given by $\epsilon = (w/g)^2/2$. It will be presumed that these systematic patterns are uncorrelated with the shear patterns on the sky.

Systematic errors are not necessarily reduced by averaging over many fields. Here, it will be assumed that the PCA analysis reduces the error in the PSF ellipticity calibration by a factor 10, based on analyses of existing Blanco data by Jain and Jarvis (2006). [This reduction is actually conservative - much better reductions have been achieved in practice.] If we designate this reduction factor by $F(= 1/10)$, we find that the contribution to the total covariance error is $\sigma_{PSF} = (w/g)^4 F^2/4$. We want this value to increase the total error by no more than 10%. This condition becomes

$$\sigma_{PSF} = \sqrt{\sigma_{CV}^2 + \sigma_{shape}} \times \sqrt{1.1^2 - 1}.$$

This is the final requirement on the optics PSF for residual whisker length after removal of a linear fit to the PSF shape. Evaluating, we find $w = 0.04$ arcsec rms.